University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar, virtual (via Zoom), December 7 and 14, 2020, 14:00-16:40

Schedule

Time	Speaker	Title
December 7		Opening at 14:00
14:00	*Opening*	
14:10	Stefano Pozza	Tridiagonalization of systems of coupled linear differential equations with variable coefficients by a Lanczos-like method
14:30	Karel Tůma	Phase-field modeling of multivariant martensitic transformation at finite-strain: computational aspects and large-scale finite-element simulations
14:50	Michal Pavelka	Towards hyperbolic electrochemistry
15:10	Marek Cúth	Lipschitz free spaces isomorphic to their infinite sums and geometric applications
15:30	*Short break*	(10 minutes)
15:40	Giovanni Gravina	Contactless rebound of elastic bodies in a viscous incompressible fluid
16:00	Malte Kampschulte	Variational approaches to dynamics: Breathing through masks and other problems
16:20	Sebastian Schwarzacher	Periodic solutions to fluid-structure interactions
December 14		Starting at 14:00
14:00	*Opening*	
14:10	Klára Kalousová	Enthalpy method for two-phase mixtures – Effect of composition
14:30	Jaroslav Haas	Dynamics of hierarchical multiples in nuclear star clusters
14:50	Josef Hanuš	V band photometry of asteroids from the All-Sky Automated Survey for Supernova
15:10	Scott Congreve	*hp*-version Recovered Finite Element Methods
15:30	*Short break*	(10 minutes)
15:40	Kathryn Lund	Stability analysis for block classical Gram-Schmidt and its Pythagorean variants
16:00	Gianmarco Sperone	On the steady motion of Navier-Stokes flows past a fixed obstacle in a three-dimensional channel under mixed boundary conditions
16:20	*Closing discussion*	

Abstracts

Scott Congreve: *hp*-version Recovered Finite Element Methods. A *recovered finite element method* is a Galerkin finite element method constructed via recovery operators over element-wise discontinuous Galerkin finite spaces. These methods have several attractive feature over classical finite element methods and discontinuous Galerkin approaches.

More formally, we consider a *recovery operator* $\mathcal{E} : V_h \mapsto \tilde{V}_h \cap H_0^1(\Omega)$ mapping a *discontinuous* piecewise polynomial V_h over a triangulation onto *continuous* piecewise polynomial space $\tilde{V}_h \cap H_0^1(\Omega)$ over the same, or finer, triangulation. We can then construct, for example, the method: find $u_h \in V_h$ such that

$$\int_\Omega \nabla\mathcal{E}(u_h) \cdot \nabla\mathcal{E}(v_h)\,\mathrm{d}x + s(u_h, v_h) = \int_\Omega f\mathcal{E}(u_h)\,\mathrm{d}x, \qquad \text{for all } v_h \in V_h,$$

to solve the Poisson problem with homogeneous Dirichlet boundary conditions with a suitable *stabilisation* $s(\cdot,\cdot)$. This method simultaneously produces both a discontinuous approximation u_h and conforming approximation $\mathcal{E}(u_h)$.

Existing work considers a uniform polynomial degree for the discontinuous and conforming finite element spaces over the triangulation. In this talk, we consider the extension of the recovered finite element method to a discontinuous space consisting of different polynomial degrees on different elements of the triangulation. This method allows for the retro-fitting of existing conforming finite element discretisations with the power of p adaptivity with minimal computational overhead.

Marek Cúth: Lipschitz free spaces isomorphic to their infinite sums and geometric applications. Given a metric space M, it is possible to construct a Banach space $F(M)$ such that the Lipschitz structure of M corresponds to the linear structure of $F(M)$. This Banach space $F(M)$ is sometimes called the "Lipschitz-free space over M". During the seminar I will talk about a recent paper with J. L. Ansorena, F. Albiac a M. Doucha, where we investigate structure of those spaces. I will concentrate on some results where the extension of Lipschitz maps from certain subsets to the whole metric space is involved.

Giovanni Gravina: Contactless rebound of elastic bodies in a viscous incompressible fluid. In this talk, we investigate the phenomenon of particle rebound in a viscous incompressible fluid environment. We focus on the important case of no-slip boundary conditions, for which it is by now classical that, under certain assumptions, collisions cannot occur in finite time. Motivated by the desire to understand this fascinating yet counterintuitive fluid-structure interaction, we introduce a reduced model for which we show that rebound is possible even in the absence of a topological contact.

Jaroslav Haas: Dynamics of hierarchical multiples in nuclear star clusters. Nuclear stars clusters are extended and extremely dense stellar systems located in the innermost parts of galaxies. It is commonly believed that they host supermassive black holes at their very centres. Presence of such massive objects provides an unique possibility to investigate the dynamics of hierarchical multiples in the complex environment of galactic nuclei. In my talk, I will focus on some of its aspects.

Josef Hanuš: V band photometry of asteroids from the All-Sky Automated Survey for Supernova. The All-Sky Automated Survey for Supernovae (ASAS-SN) project that currently consists of 24 small-aperture telescopes distributed around the globe is automatically surveying the entire visible sky every night down to about $V \sim 18mag$. Although primarily designed for supernovae detection (or any kind of transients in general), asteroids are common intruders in the ASAS-SN's fields of view. We extracted V-band photometry from the ASAS-SN images for $> 30,000$ asteroids that get brighter than $V \sim 17mag$. The data span years 2012-2019 and sample up to 7 consequent apparitions for each asteroid. We analyzed the photometric data in the means of the lightcurve inversion method and derive rotation periods, spin axis directions and shapes for a sample of studied asteroids.

Klára Kalousová: Enthalpy method for two-phase mixtures – Effect of composition. Outer solar system moons have thick ice layers where melting may occur. Apart from ice and water, other compounds are likely present, such as salts or ammonia. In order to properly describe the thermal evolution of such a system, the mixture composition has to be addressed and solved for. To test our formulation and implementation, we compare our solution with the two-phase convection benchmark. This is a joint project with Ondřej Souček.

Malte Kampschulte: Variational approaches to dynamics: Breathing through masks and other problems. We show how newly developed variational methods (along the lines of [Benešová, K., Schwarzacher 2020]) can be used to prove existence of weak or measure-valued solutions to large classes of nonlinear, dynamical problems of possibly mixed Lagrangian/Eulerian type in continuum mechanics. Since the main focus of these methods is the energy balance, they allow us to base our modelling directly on that. As an illustrative example we will apply this method to the topical problem of an incompressible fluid flow interacting with a porous, nonlinear elastic solid.

Kathryn Lund: Stability analysis for block classical Gram-Schmidt and its Pythagorean variants. The block version of classical Gram-Schmidt (BCGS) is often employed to efficiently compute orthogonal bases for Krylov subspace methods and eigenvalue solvers, but a rigorous proof of its stability behavior has not yet been established until this work. It is shown that the usual implementation of BCGS can lose orthogonality like $O(\varepsilon)\kappa^{p-1}(X)$, where p is the number of block partitions of X and ε is the unit round-off. A useful intermediate quantity denoted as the Cholesky residual is given special attention and, along with a block generalization of the Pythagorean theorem, this Cholesky residual is used to develop more stable variants of BCGS. These variants are shown to have $O(\varepsilon)\kappa^2(X)$ loss of orthogonality with relatively relaxed conditions on the intra-block orthogonalization routine. A variety of numerical examples verify the theoretical bounds.

Michal Pavelka: Towards hyperbolic electrochemistry. Electrochemistry concerns about transport of reacting charged matter in coupling with electrodynamics and heat, and it is necessary for the development of fuel cells and batteries. The evolution equations that describe most electrochemical processes are typically of parabolic (diffusion-like) nature. Such parabolic equations can be seen as an approximation of more detailed hyperbolic equations coming from Hamiltonian continuum mechanics, which cover a much wider range of ultrafast wave-like phenomena in mutual coupling with electrochemical reactions.

We have recently suggested a possibility for the construction of such detailed equations, but many questions remain open, including validity and generality of the equations, characterization of newly observed phenomena, their exploitation within electrochemistry, and appropriate numerical methods. The hope is to bring up a new class of highly efficient electrochemical devices based on interactions between the ultrafast hyperbolic processes. This is joint work with P. Vágner and O. Esen.

Sebastian Schwarzacher: Periodic solutions to fluid-structure interactions. The lecture will be about a joint project with C. Mindrila on the subject of the existence of periodic solutions in fluid-structure interactions. We couple an elastic (non-dissipative) solid shell with the incompressible Navier Stokes equation. We consider this to be a first step towards the mathematical description of some very relevant applications in this field, as the periodic blood-flow in a vessel driven by the periodic force of a heart beat.

Difficulties arise due to the fact that the geometry for the fluid domain is a part of the solution. These are overcome by regularity estimates that are beyond energy estimates. For that we developed a three step testing procedure.

Stefano Pozza: Tridiagonalization of systems of coupled linear differential equations with variable coefficients by a Lanczos-like method. Under certain regularity assumptions, any system of coupled linear differential equations with variable coefficients can be tridiagonalized by a time-dependent Lanczos-like method. The proof we present formally establishes the convergence of the Lanczos-like algorithm and yields a full characterization of algorithmic breakdowns. This is a key piece in evaluating the elusive ordered exponential function both formally and numerically.

Gianmarco Sperone: On the steady motion of Navier-Stokes flows past a fixed obstacle in a three-dimensional channel under mixed boundary conditions. We analyze the steady motion of a viscous incompressible fluid in a three-dimensional channel containing an obstacle through the Navier-Stokes equations with mixed boundary conditions: the inflow is given by a fairly general datum and the flow is assumed to satisfy a constant traction boundary condition on the outlet, together with the standard no-slip assumption on the obstacle and on the remaining walls of the domain. Explicit bounds on the inflow velocity guaranteeing existence and uniqueness of such steady motion are provided after estimating some Sobolev embedding constants and constructing a suitable solenoidal extension of the inlet velocity through the Bogovskii formula.

A quantitative analysis of the forces exerted by the fluid over the obstacle constitutes the main application of our results: by deriving a volume integral formula for the drag and lift, explicit upper bounds on these forces are given in terms of the geometrical constraints of the domain.

Karel Tůma: Phase-field modeling of multivariant martensitic transformation at finite-strain: computational aspects and large-scale finite-element simulations. Large-scale 3D martensitic microstructure evolution problems are studied using a finite-element discretization of a finite-strain phase-field model. The model admits an arbitrary crystallography of transformation and arbitrary elastic anisotropy of the phases, and incorporates Hencky-type elasticity, a penalty-regularized double-obstacle potential, and viscous dissipation. The finite-element discretization of the model is performed in Firedrake and relies on the PETSc solver library. The large systems of linear equations arising are efficiently solved using GMRES and a geometric multigrid preconditioner with a carefully chosen relaxation. The modeling capabilities are illustrated through a 3D simulation of the microstructure evolution in a pseudoelastic CuAlNi single crystal during nano-indentation, with all six orthorhombic martensite variants taken into account. Robustness and a good parallel scaling performance have been demonstrated, with the problem size reaching 150 million degrees of freedom. This is joint work with M. Rezaee-Hajidehi, J. Hron, P.E. Farrell, and S. Stupkiewicz.